Quantitative techniques

The material contained in this chapter has been compiled to provide you with the background you will need to answer quantitative questions in your IB Economics exams. It is intended for students studying economics at both standard and higher levels.

Students sometimes come to the study of economics concerned that the course may be based on mathematical techniques they might be unable to handle. However, as you will discover in the pages that follow, the mathematics you need to do well in your IB course is quite limited. The few simple techniques you will find here repeat themselves in a variety of applications in your IB course. Most if not all of these techniques involve mathematical ideas or methods you have very likely already encountered in your earlier years as a student or as part of your IB studies. Therefore, most of what is included in this chapter will be simply a review for you.

Note that answers to all the 'Test your understanding' questions appear in the teacher's resource.

1 Percentages and percentage changes

Percentages

Why percentages are important

'Percent' means 'out of 100'. It is simply another way of expressing a fraction or a decimal (which is also a kind of fraction) as a number in relation to 100.

Percentages allow us easily to make comparisons between fractions that cannot otherwise be easily compared. Suppose you take two tests, and you score $\frac{17}{20}$ in one and $\frac{29}{35}$ in the other. In which test did you get the higher score? There are only two ways to answer this question. One is to the find the lowest common denominator of the fractions, which is 140, and express the first score as $\frac{119}{140}$ and the second as $\frac{116}{140}$. This shows you scored better in the first test.

A much simpler way, however, is to express the two scores as percentages, which are 85% for the first and 82.8% for the second. The use of percentages is not only a much easier method, but it also allows you to compare all your other test scores with each other.

Percentages in relation to fractions and decimals

Changing a fraction and a decimal into a percentage

Let's say we would like to express a fraction, such as $\frac{34}{75}$, as a percentage:

i We divide the numerator by the denominator, which converts the fraction into a decimal.
ii We multiply by 100%:

$$\frac{34}{75} = 0.4533; \qquad 0.4533 \times 100\% = 45.33\%$$

Changing a percentage into a fraction

To change a percentage into a fraction, we do the following:

i divide the percentage by 100 and remove the % sign
ii simplify the resulting fraction.

For example:

$$50\% = \frac{50}{100} = \frac{1}{2} \qquad 78\% = \frac{78}{100} = \frac{39}{50}$$

$$175\% = \frac{175}{100} = \frac{7}{4}$$

Changing a percentage into a decimal

We divide the percentage by 100 (move the decimal point two places to the left) and remove the % sign:

50% = 0.50 78% = 0.78 175% = 1.75

Using percentages: examples

Example 1: Convert the test scores noted earlier, $\frac{17}{20}$ and $\frac{29}{35}$, into percentages.

$$\frac{17}{20} \times 100\% = 0.850 \times 100\% = 85.0\%$$

$$\frac{29}{35} \times 100\% = 0.829 \times 100\% = 82.9\%$$

(Note that when we multiply by 100%, we simply move the decimal point two places to the right and add the % sign.)

Example 2: In 2009, the total population of a country called Mountainland was 46.3 million. The rural population (living outside of cities) of Mountainland in the same year was 17.7 million. What percentage of Mountainland's population was rural?

We express the rural population as a fraction of the total population, and convert this fraction into a percentage:

$$\text{\% of population that is rural} = \frac{17.7\ \textit{million}}{46.3\ \textit{million}} \times 100\%$$

$= 0.3823 \times 100\% = 38.23\%$

Example 3: In a country called Flatland, 22 million people live in cities and 38 million live in the countryside. What percentage of Flatland's population lives in cities?

The total population is:

22 million + 38 million = 60 million

Therefore, the percentage of people in cities is:

$$\frac{22}{60} \times 100\% = 36.7\%$$

Example 4: 12% of a graduating class of 150 students plan to study economics at university. How many students plan to study economics?

When we want to find a number that corresponds to a percentage of another given number, we multiply the given number by the percentage in decimal form:

$12\% = 0.12$; $0.12 \times 150 = 18$ students

Example 5: 70% of 140 students are actively involved in sports. How many students are involved in sports?

$70\% = 0.70$; $0.70 \times 140 = 98$ students

TEST YOUR UNDERSTANDING 1

1 Change the following decimals into percentages:

- a 0.573
- b 0.628
- c 1.247
- d 0.645

2 Change the following fractions into percentages:

- a $\frac{3}{9}$
- b $\frac{271}{977}$
- c $\frac{175}{65}$
- d $\frac{5}{178}$

3 Change the following percentages into decimals:

- a 24.5%
- b 25%
- c 99%
- d 125%

4 In a class of 25 students, 15 came to school by bus, and the rest walked to school.

- a What percentage of students came by bus?
- b What percentage of students walked?

5 A business makes a profit of 25% of its sales of $17 000. How much profit does it make?

6 In 2009, 14% of Mountainland's population (of 46.3 million) were university graduates. How many people does this correspond to?

Percentage changes

Students studying economics at both SL and HL should be able to interpret percentage changes as well as perform calculations in exams.

Given two numbers, how to calculate a percentage change

To calculate a percentage change, we must have an initial number and a final number. The percentage change expresses the change (an increase or a decrease) as a percentage of the initial number. Suppose we want to find the percentage change from 50 to 75. We express the change as a fraction of the initial number and then convert this into a percentage. The change is 25 (=75−50), and the initial number is 50. Therefore, we have $\frac{25}{50} = 0.50$, which is equivalent to 50%.

> More generally, we can calculate a percentage change in a variable, A, by using the following formula:
>
> % change in
>
> $$A = \frac{\textit{final value of A} - \textit{initial value of A}}{\textit{initial value of A}} \times 100\%$$

A *percentage increase or percentage decrease* is shown by whether the percentage change that is calculated by use of this formula is a positive or negative number.

Example 6: Suppose the population of Mountainland was 45.7 million in 2008 and 46.3 million in 2009. What was the percentage change in population from 2008 to 2009?

Applying the formula, we have:

% change in population

$$= \frac{46.3\ \textit{million} - 45.7\ \textit{million}}{45.7\ \textit{million}} \times 100$$

$$= \frac{0.6\ \textit{million}}{45.7\ \textit{million}} \times 100\%$$

= 0.013 × 100% = 1.3%

The population of Mountainland thus *increased* by 1.3% (1.3 is a positive number). 1.3% is a *percentage increase*.

Example 7: Now suppose that the rural population of Mountainland was 18.3 million in 2008 and 17.7 million in 2009. What was the percentage change?

% change in rural population

$$= \frac{17.7\ \textit{million} - 18.3\ \textit{million}}{18.3\ \textit{million}} \times 100\%$$

$$= \frac{-0.6\ \textit{million}}{18.3\ \textit{million}} \times 100\%$$

= −0.033 × 100% = −3.3%

The rural population of Mountainland therefore *decreased* by 3.3% (−3.3 is a negative number).

Example 8: The data in the table below show Mountainland's real GDP (real output produced) for the period 2008–2010. Calculate the rate of growth in real GDP in (a) 2008–2009, and (b) 2009–2010.

Year	Real GDP (in trillion Mnl, the national currency)
2008	5.6
2009	5.7
2010	5.5

A percentage change may sometimes be referred to as 'rate of growth', which may be positive or negative.

a Rate of growth in real GDP, 2008–2009:

% growth in real GDP (2008–2009)

$$= \frac{5.7\ \textit{trillion} - 5.6\ \textit{trillion}}{5.6\ \textit{trillion}} \times 100\%$$

$$= \frac{0.1\ \textit{trillion}}{5.6\ \textit{trillion}} \times 100\%$$

= 0.018 × 100% = 1.8%

Mountainland experienced a positive rate of growth in real GDP of 8% in 2008–9.

b Rate of growth in real GDP, 2009–10:

% growth in real GDP (2009–2010)

= −0.035 × 100% = −3.5%

Mountainland experienced a negative rate of growth in real GDP of −3.5% in 2009–2010.

> In practice, when multiplying a decimal by 100% to convert it into a percentage, we drop the '%' so it looks like we are multiplying the decimal by '100'.

Given a number and its percentage change, how to calculate the change and the final number

Example 9: Due to a combination of a higher birth rate and a large influx of immigrants into Mountainland, the population increased by 4.8% in the period 2009–2010. The size of the population was 46.3 million in 2009. (a) How many people were added to Mountainland's population in 2009–2010? (b) What was the size of the population in 2010 as a result of the 4.8% increase?

a The number of people added is 4.8% of the whole population in 2009 (46.3 million). Therefore:
4.8% × 46.3 million = 0.048 × 46.3 million
= 2.2 million people

b We can find the 2010 population in two ways:

- **i** Add the increase of 2.2 million to the initial population of 46.3 million:
  2.2 million + 46.3 million
  = 48.5 million in 2010
- **ii** A more direct way is to begin with the initial population of 46.3 million, and multiply this by 1 plus the percentage change in decimal form, or 1 + 0.048 = 1.048:
  population in 2010 = 46.3 million × 1.048
  = 48.5 million people in 2010

Note that the answers obtained in the two ways are identical.

Example 10: The rural population in Mountainland, which was 17.7 million in 2009, fell by 3.8% in 2009–2010. What was the size of the rural population in 2010?

Again, we can do this in two ways:

- **i** Find by how much the rural population fell, and subtract this from the rural population in 2009:
  rural population decrease 2009–2010 = 3.8% × 17.7 million = 0.038 × 17.7 million
  = 0.7 million people
  rural population in 2010 = 17.7 million − 0.7 million = 17.0 million people
- **ii** In a more direct way, we take the initial population of 17.7 million, and multiply this by 1 minus the percentage change in decimal form, or 1 − 0.038
  = 0.962
  rural population in 2010 = 17.7 million × 0.962
  = 17.0 million people

Note that the answers obtained in the two ways are identical.

TEST YOUR UNDERSTANDING 2

1. You are interested in buying a book that cost 30 Mnl, but discover that its price has increased by 20%. What is the book's new price?
2. Riverland's GDP of 259 Rvl billion in 2009 grew to 272 Rvl billion in 2010 but then fell to 267 Rvl billion in 2011. Calculate the rate of growth in Riverland's GDP in the period
   - **a** 2009–2010, and
   - **b** 2010–2011.
3. In 2005, Riverland had a population of 32.9 million people. In the period 2005–2010, its population grew by 7.2%. What was its population in 2010?
4. It is estimated that Flatland's unemployed workers were 1.2 million in 2009. By 2010, the number of unemployed had fallen to 1 million. What was the percentage change in unemployed workers over this period?
5. A company had profits of $2.5 million in 2009. It insists on an 8.0% increase in profits per year. What will its profits be in 2010 if it meets its goal?

2 Understanding and interpreting graphs and diagrams

Graphs and diagrams are simply 'pictures' showing how variables change and how they are related to each other. Sometimes, the information contained in graphs and diagrams can be described in words; however, 'pictures' allow us to see and understand information far more quickly and effectively. Therefore, graphs and diagrams are very important in presenting real-world information as well as in building and presenting economic theories and models.

Graphs that display information about a single variable

A variable is a measure of something that can take on different values; it is something that 'varies' or 'changes'. Very often, we are interested in studying changes in a single variable. We can do this by using three types of graph: pie charts, bar graphs and line graphs. Each one serves different purposes (though there are also some overlaps).

Pie charts

Pie charts are convenient to use when we want to show how a whole is divided up between different parts. The 'raw data' appearing in Table 1 present how the total population in the world in 2008 was divided between the countries in the world according to income levels. The variable is population, which varies according to group of countries. The last column shows the percentage of the world's population that was in each income group. 'High income' countries are those considered to be economically more developed, while 'upper middle income', 'lower middle income' and 'low income' countries are considered to be economically less developed.

This information is presented as a pie chart in Figure 1. The pie chart is derived by multiplying the 360° of a circle by the same percentage that appears for each income level in the table, thus obtaining the 'slice' of the 'pie' that corresponds to each income level. We can see straightaway that a far larger percentage of the world's population, or 84% (= 14.2% + 55.3% + 14.5%) lives in economically less developed countries, compared to only 16% who live in more developed ones. Note that Table 1 and Figure 1 display exactly the same information, yet it is much easier to 'read' it in the pie chart.

Income level	Population (millions)	% of total
High income	1068.5	16.0
Upper middle income	948.5	14.2
Lower middle income	3702.2	55.3
Low income	972.8	14.5
Total	6692.0	100.0

Source: *The World Bank, Data Catalog (http://data. worldbank.org/ data-catalog).*

Table 1: World population (in millions) and distribution among countries grouped by income levels, 2008

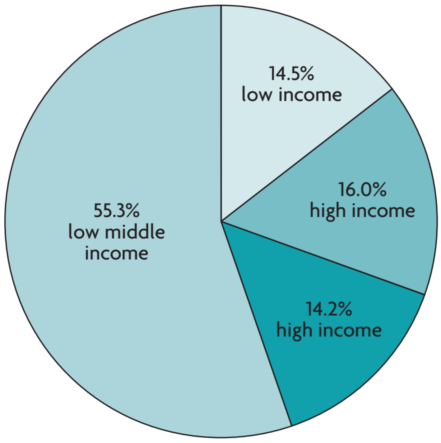


Figure 1: World population (millions) and distribution among countries grouped by income levels, 2008

Source: *The World Bank, Data Catalog (http://data. worldbank.org/ data-catalog).*

Bar graphs

The information in Table 1, presented in a pie chart in Figure 1, can also be presented as a bar graph, as shown in Figure 2. In Figure 2(a), percentages of the world's population are measured along the vertical axis, and the income groups appear along the horizontal axis. Figure 2(b) is the same as Figure 2(a) except that the axes have been reversed. Both presentations are used equally effectively.

However, comparing the pie chart in Figure 1 with the two bar graphs in Figure 2, we can see that the pie chart provides a more effective representation of the different shares of the world's population, because it allows the viewer to see more clearly the relative size of each share of the whole.

Bar graphs, on the other hand, are very useful in providing a visual representation of other kinds of information and data that cannot be shown in a pie chart. They are very convenient for illustrating **cross-section** data, which are the values taken by a single variable at a particular time (such as a year) for different groups in a population. The bar graph in Figure 3(a) measures on its vertical axis the percentage of children of primary school age who are enrolled in school, against the countries of the four income groups, which appear on the horizontal axis, in 2007. This graph tells us that 95% of children in high-income countries attend primary school, compared to 77.6% in low-income ones, and so on for the other two groups and the world as a whole.

Bar graphs need not measure percentages on the vertical axis. The bar graph in Figure 3(b) shows the number of tourist arrivals per 1000 people in a given year for selected countries. For example, in Cyprus,

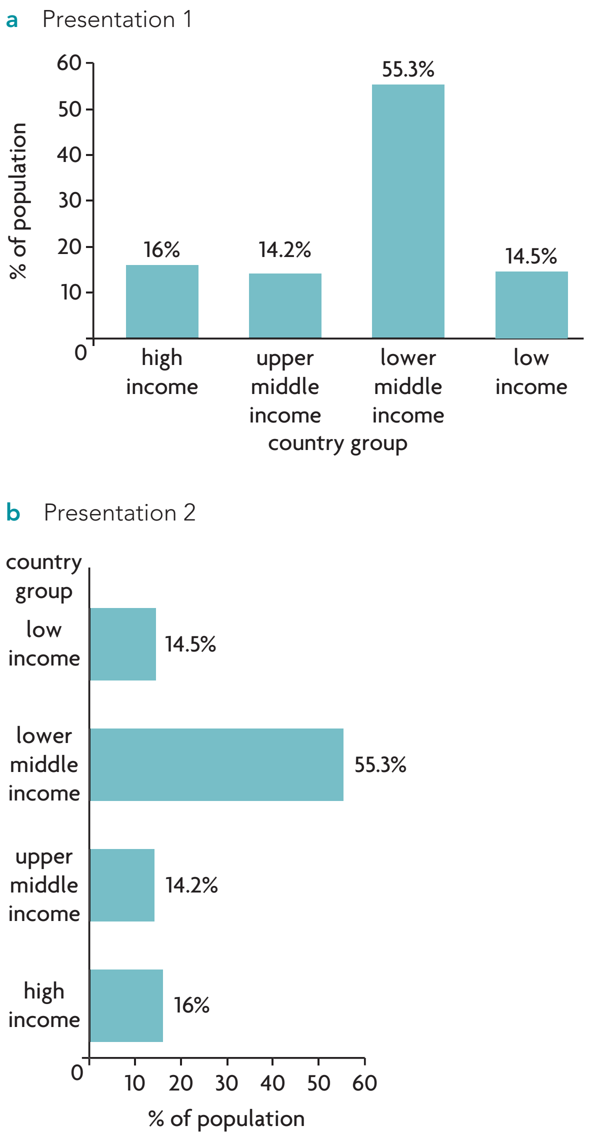


Figure 2: Bar graphs based on the information in Table 1

there are 2676 tourists per year for every 1000 local residents, in Saint Kitts and Nevis there are 2259, and so on for the other countries shown.

Bar graphs are also useful in presenting how a variable changes for each group from one time period to another. This can be seen in Figure 3(c), which shows the rate of growth of GDP for three years and for seven country groups. This kind of graph allows us to see not only how the variable, in this case the rate of growth in GDP, varies from region to region (highest in countries of East Asia and Pacific and lowest in high income countries), but also, we can see that for all country groups, the rate of growth of GDP was lower in 2008 compared to 2007, and was in some cases lower than in 2006.

Some variables illustrated in bar graphs may sometimes take on negative values, that is, the bars can fall below the horizontal axis. Whether or not this can happen depends of course on the nature of the variable (it could not happen in Figure 3(b) because there cannot be a negative number of tourists). Figure 3(d) shows the current account balance as a percentage of GDP in selected countries (we will study the current account balance in Chapter 14 of the textbook). Very briefly and simply (for the time being), the current account balance is a measure of inflows and outflows of money in a country for particular purposes. If inflows are greater than outflows, there is a positive balance, which appears as a bar above the horizontal axis. If inflows are smaller than outflows, there is a negative balance and this appears as a bar below the horizontal axis.

Line graphs

Line graphs are a convenient way to represent a variable that takes on different values over time. We have seen that this can to some extent also be shown in a bar graph, as in Figure 3(c). However, if we are interested in seeing how a variable changes over an extended period, and also want to easily make comparisons, a line graph is more appropriate. A line graph typically measures time (in months, years, etc.) on the horizontal axis, while the variable that is being examined appears on the vertical axis. Line graphs showing change over time use **time-series** data which are data for a variable that changes over time.

Figure 4(a) shows how the unemployment rate (the percentage of unemployed people out of the total labour force) varies over time in the European Union (EU) as well as in two EU countries. The graph covers a period of 28 years, and is very effective in illustrating the fluctuations in the rate of unemployment. In addition, it allows us to make comparisons in unemployment rates of individual countries and the EU, whose unemployment rate is an average over all its members. As we can see, Cyprus's unemployment rate has been lower and Belgium's higher (until about 2003–2004) than the average over the EU.

The variables illustrated in line graphs can also sometimes take on negative values, as in the case of bar graphs. Figure 4(b) shows the annual rate of growth of agricultural output (per person) in sub-Saharan Africa for the period 1968–2004. In some years this has been positive (above the horizontal axis) while in others it has been negative (below the

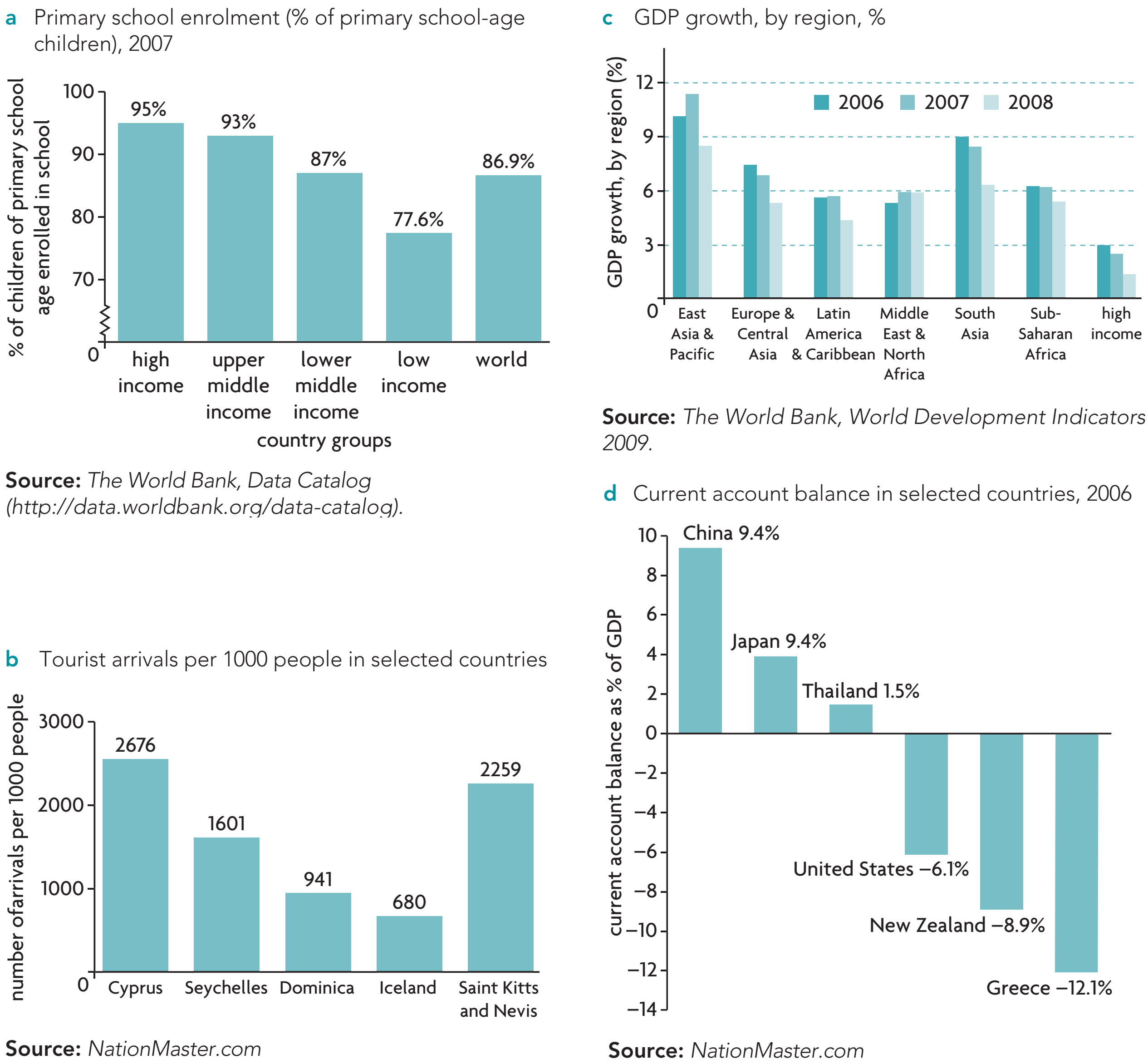


Figure 3: Bar graphs (cross-section data)

horizontal axis). As you may remember from the discussion above, a positive rate of growth means that output is *increasing*, while a negative rate of growth indicates *decreasing* output.

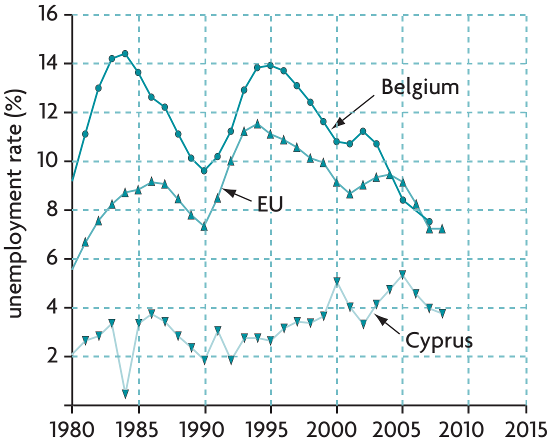
TEST YOUR UNDERSTANDING 3

Look through local newspapers and magazines and find examples of pie charts, bar graphs and line graphs. Which of these display cross-section data; which display time-series data? Explain, in a general way, what each graph illustrates, and describe any patterns or trends you detect.

Graphs displaying the relationship between two variables

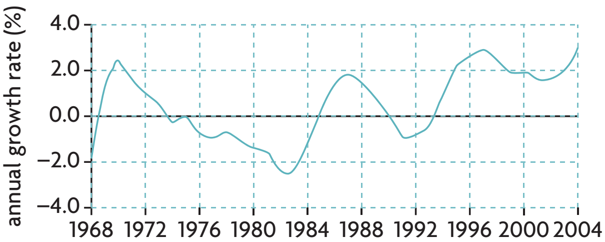
Pie charts, bar graphs and line graphs that display information about a single variable are usually constructed in order to present real world data in an organised and logical way, allowing viewers to easily make sense of the information and if possible, detect patterns that increase our understanding of the complicated world around us. Yet very often we want to go beyond a description of the world offered by these graphs in order to discover *how variables are related to each other*. To do this, we must use graphs that focus on two

a Unemployment rate in the EU and selected EU countries



Source: *World Health Organization/Europe, Health for all database, January 2010.*

b Agricultural output growth per person in sub-Saharan Africa



Source: *The World Bank, World Development Report, 2008.*

Figure 4: Line graphs (time-series data)

interrelated variables. These kinds of graphs are very important in illustrating economic theories and building economic models.

Constructing and interpreting graphs that relate two variables to each other

Graphs that illustrate how variables are related to each other measure a different variable on each axis. Each axis on the graph represents a number line, which measures units of the variable. Figure 5(a) presents a vertical and a horizontal number line. The vertical number line begins with negative numbers at the bottom end, which increase as we move upward until they reach zero, and then become positive. The horizontal number line begins with negative numbers at the left, which increase as we move rightward until they reach zero, and then become positive. In both number lines, the number 0 is called the *origin*. In each number line, the numbers represent units of the variable that is being measured. Therefore, the value of the variable measured on the vertical number line increases in the upward direction, and the value of the variable measured on the horizontal number line increases in the rightward direction.

To construct a graph, we put the vertical and horizontal number lines together, so that they are perpendicular to each other and intersect at the origin of each one, as shown in Figure 5(b). Each number line in the graph is referred to as an axis. By convention, the horizontal axis is sometimes called the x-axis and the vertical is called the y-axis. The four spaces that are carved out by the intersecting axes are called *quadrants*, and are numbered from I to IV. Quadrant I represents combinations of two positive numbers, quadrants II and IV represent combinations of one positive and one negative number, and quadrant III represents combinations of two negative numbers.

In economics, most of the relationships we examine involve combinations of two positive numbers, in which case we simplify the graph by considering only the first quadrant, ignoring the remaining three. This is shown in Figure 5(c). However, sometimes we may want to examine relationships involving one positive and one negative number, in which case we may consider a graph such as in Figure 5(d) or (e).

Positive (direct) relationships between two variables

Each point on a graph is specified by two numbers, one from each of the two axes. Suppose we are examining the relationship between the following two variables: the number of calories consumed per day and the amount of weight (in kilograms, abbreviated as kg) that will be gained in one month. The data on these two variables are presented in Table 2 below, and are graphed (or plotted) in Figure 6(a) where daily caloric consumption is measured on the horizontal axis and monthly weight gain on the vertical axis. Each pair of points in the table corresponds to a point in the graph. For example, point e corresponds to 2500 calories per day, and to 2 kg of weight gain per month. On the graph, this point is found by drawing a line upward from 2500 on the horizontal axis, and also drawing a horizontal line from 2 on the vertical axis; point e is where the two lines meet. Every other point on the graph is plotted in the same way.

Each point in the graph is represented as: (h, v), where h= takes on the value of the variable measured on the horizontal axis and v= takes on the value of the variable measured on the vertical axis. Therefore, point e is represented as (2500,2); point g as (3000,4).

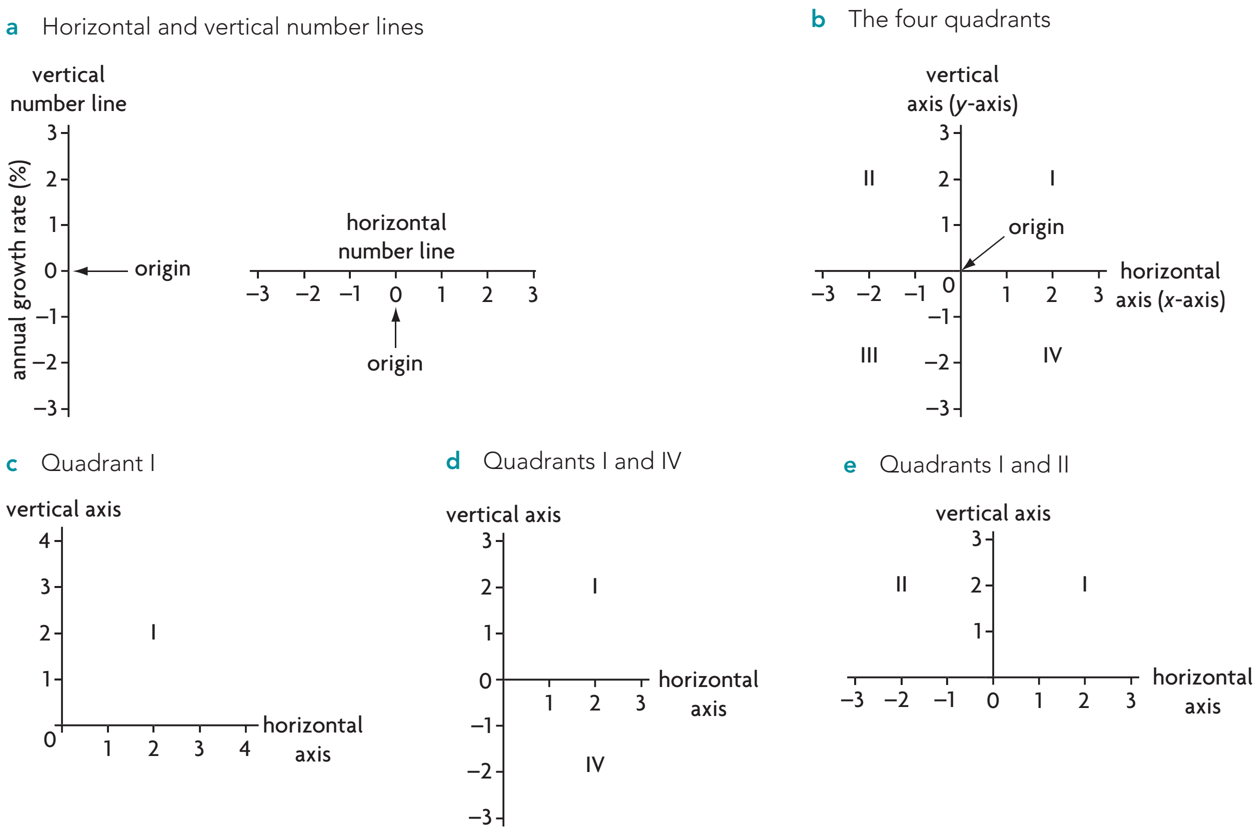


Figure 5: Number lines and quadrants

> Each point in a graph can be represented as (h, v), where h = the value of the variable on the horizontal axis and v = the variable on the vertical axis.

Drawing a line through all the points gives a straight line, which is called a **curve**. All lines in graphs (and diagrams) are referred to as curves, regardless whether they are straight or curved.

Note that the horizontal axis in Figure 6 contains a squiggly part close to the vertical axis. The reason is that the numbers on this axis jump from 0 to 1500 in a very short space, whereas all other points have an identical distance between them, each space representing 250 calories. The squiggly line means that some numbers of the number line of this axis have been skipped.

Once a line is drawn connecting the points in a graph, it is possible to read off other combinations of the two variables. For example, a point exactly in between points e and f on the curve corresponds to the points between the values of the two variables on each of the axes, which are 2.5 kg and 2625 calories.

Daily caloric consumption	Monthly weight gain (kg)	Point on graph (Figure 6a)
1500	−2	a
1750	−1	b
2000	0	c
2250	1	d
2500	2	e
2750	3	f
3000	4	g

Table 2: The relationship between calories consumed daily and weight gain per month

The graph shows that consumption of 2000 calories per day results in a steady weight, as weight gain is 0 kg. If calories consumed rise above 2000 per day, the result is weight gain, while if they are less, the result is weight loss (negative 'weight gain' is equivalent to weight loss).

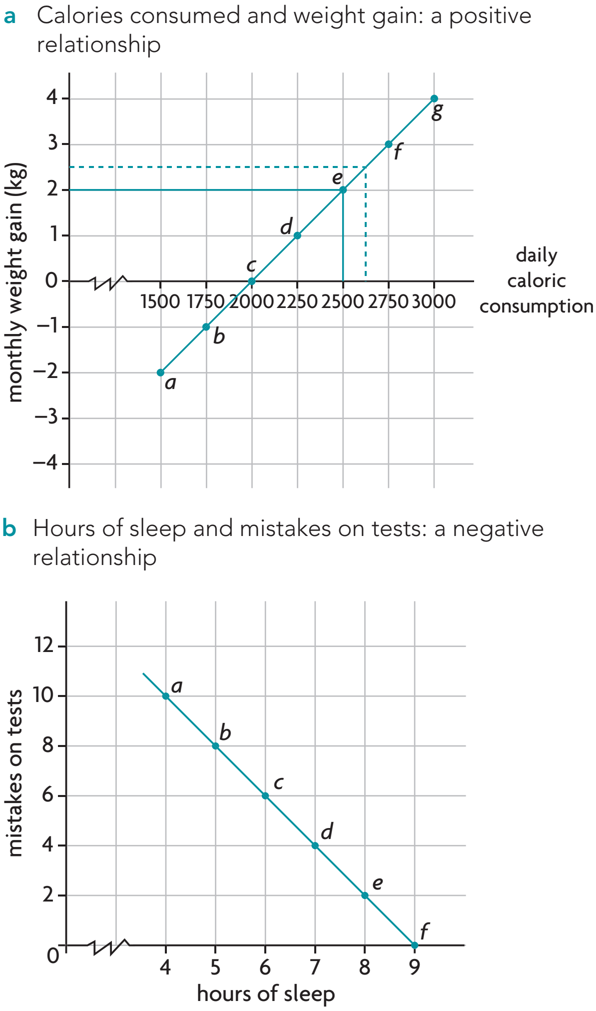


Figure 6: Positive and negative relationships between two variables

The curve of Figure 6(a) shows that the two variables we are examining are related in a particular way: as daily consumption of calories increases, weight gain also increases. This is called a **positive**, or **direct** relationship:

A positive (or direct) relationship between two variables is illustrated by a curve that moves upward and to the right, showing that as one variable increases, the other also increases. Alternatively, if one variable decreases, the other also decreases. In a positive relationship, the two variables change in the same direction.

Negative (or indirect) relationships between two variables

Let's now consider a different kind of relationship, using the information of Table 3, which provides data on two variables: number of hours of sleep and number of mistakes on tests. Each pair of data corresponds to a single point in the graph that appears in Figure 6(b). This graph is plotted entirely in quadrant I, with positive values of both variables, as it is not possible to have a negative number for hours of sleep or for mistakes on tests. This graph shows that few hours of sleep are associated with a larger number of mistakes on tests, while more hours of sleep mean fewer mistakes on tests. This is called a **negative**, also known as an **indirect**, relationship:

Number of hours of sleep	Number of mistakes on tests	Point on graph (Figure 6b)
4	10	a
5	8	b
6	6	c
7	4	d
8	2	e
9	0	f

Table 3: The relationship between hours of sleep and mistakes on tests

A negative (or indirect) relationship between two variables is illustrated by a curve that moves downward and to the right, showing that as one variable increases, the other variable decreases. In a negative relationship, the two variables change in opposite directions.

Graphs and the cause-and-effect relationship between two variables

One important reason why we construct and study two-variable graphs is that we are trying to discover **causal relationships** between variables. A causal relationship is a 'cause-and-effect' relationship, where changes in one variable are seen as causing changes in the other variable. The variable that initiates the change is called the **independent variable**, and the variable that is influenced or affected as a result is called the **dependent variable**. A causal relationship is called a **functional relation**, where the dependent variable is a function of the independent variable. Two-variable graphs usually (though not always) display such functional relations.

In Figure 6(a), the independent variable (or the 'cause') is the daily consumption of calories, and the dependent variable (or the 'effect') is monthly weight gain. Monthly weight gain *depends* on consumption of calories, hence is the dependent variable. Figure 6(a) shows a **positive causal relationship**. In Figure 6(b), the independent variable is the number of hours of sleep; in this relationship, mistakes on tests *depend* on hours of sleep. In Figure 6(b), we see a **negative causal relationship**.

Two-variable graphs usually represent a causal relationship between the variables, where the independent variable is the 'cause' and the dependent variable is the 'effect'. Causal relationships are very important for making hypotheses, constructing theories that try to explain events and as building blocks of models that illustrate theories.

TEST YOUR UNDERSTANDING 4

For each of the following pairs of variables, explain (i) whether there is likely to be a positive or negative causal relationship between them, and (ii) which is the dependent and which is the independent variable:

- **a** income and saving
- **b** number of DVDs purchased and price of DVDs
- **c** level of salary and number of years of working experience
- **d** the temperature and number of swimmers on the beach.

Causation versus association (correlation)

Two variables may sometimes appear to be related to each other, and yet there may not be a causal relationship between them. There are three important cases leading to difficulties.

Case 1: We may observe that a society has a large number of doctors (*per capita* or per person in the population) and it also has high rates of certain diseases. We could conclude that the high rate of diseases has given rise to a large number of doctors (though we could also conclude the opposite). But would this be a valid argument? It is very possible (and likely) that each of these (the large number of doctors and the high rate of diseases) has separate, independent causes, and the apparent relation between the two is coincidental.

Case 2: A second possible difficulty is illustrated by the following simple example. Runny noses are observed to appear together with sore throats. We therefore conclude that a runny nose causes a sore throat (or vice versa). But this is clearly nonsense. Both runny noses and sore throats are caused by another (third) factor, which is usually a virus. In this case, the association of runny noses and sore throats is usually not coincidental, but the two are not causally related to each other.

Case 3: A third difficulty arises in the event that there may be a causal relationship between two variables, but we cannot be sure which causes which. For example, it is generally observed that people with more education also have higher incomes. It may therefore be supposed that more education allows people to get better jobs and therefore earn higher incomes. However, the opposite is also possible, as people with higher incomes have greater possibilities to become educated. So which causes which?

In the first two cases, there is an **association** (in mathematical terms this is called a **correlation**) between the two variables, but with no causation. In the first case, this is due to coincidence; in the second, it is due to there being another causal factor which affects both variables. In the third case, the association (or correlation) may well be due to causation, but we cannot be sure which is the causal factor.

The cases of demand and supply

In your study of economics, you will encounter many kinds of relationships between variables that are illustrated as graphs or diagrams. Two very important relationships you will make great use of are those of demand and supply.

Demand

Demand as a relationship between price and quantity demanded

Demand involves the relationship between the price of a good and the quantity of the good consumers want to buy. Table 4 shows the quantity of pizzas that the residents of Olemoo want to buy at different possible prices (in Olemooan $) each week. The data in Table 4 are graphed (or plotted) in Figure 7, where we see that there is a negative (or indirect) relationship between price and quantity: the higher the price, the fewer the pizzas that Olemooans want to buy. The quantity of pizzas Olemooans want to buy at each price is called **quantity demanded**.

Note that of the two variables we are considering, price and quantity demanded, price is the independent variable and quantity demanded is the dependent variable. This is because the quantity of pizzas Oloemooans want to buy *depends* on the price of pizzas. Therefore, there is a **negative causal relationship** between price and quantity demanded. According to correct mathematical practice, the independent variable is plotted on the horizontal axis and the dependent variable on the vertical axis (as in the cases of Figures 6 (a) and (b) above). However, many economics graphs do not follow the customary mathematical practice. *In the case of demand, the independent variable 'price' always appears on the vertical axis, while the dependent variable, 'quantity', always appears on the horizontal axis.*

Price per kg (Olemooan $)	Quantity demanded (thousand kg per week)
1	9
2	7
3	5
4	3
5	1

Table 4: Demand for pizzas by Olemooans

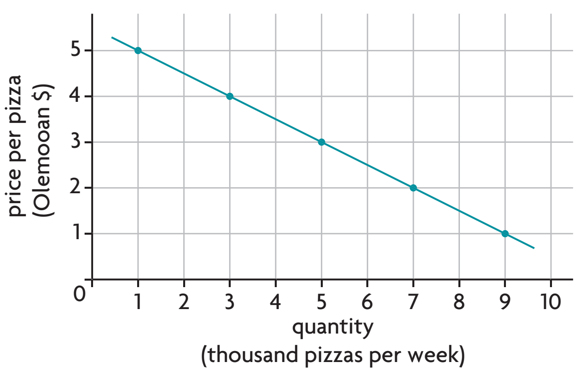


Figure 7: Demand curve: price of pizzas and quantity demanded

Shifts of the demand curve and the ceteris paribus assumption

So far, we have studied the relationship between two variables only. However, in the real world, a dependent variable usually depends on more than just one independent variable. For example, the amount of pizzas that Olemooans want to buy very likely depends not only on the price of pizzas but also on Olemooans' income, Olemooans' tastes (how much they like pizza), the number of people in the Olemooan population, and other factors. Taking income as an example, it is likely that as Olemooans' income increases, they will want to buy more pizzas. Yet this complicates matters. What if income increases, and at the same time the price of pizzas also increases? Will Olemooans want to buy more or fewer pizzas?

We now have three variables, but with the possibility of showing only two of these at the same time in a graph of a demand curve. To resolve this problem, we plot the relationship between price and quantity demanded *on the assumption that income (plus all other things that can affect demand for pizzas) is constant or unchanging. This is called the ceteris paribus assumption* (explained also in Chapter 1 of the textbook, Section 1.4), which means that all other things that can affect a relationship between two variables are assumed to be constant or unchanging.

To deal with the likely effects of income on pizzas demanded in a graph, we use the information in Table 5, which contains data on quantities of pizzas demanded for different levels of income. The data in the column indicating 'middle income', are the same as those used to plot the demand curve in Figure 7.

Figure 8 graphs the demand curves corresponding to each of the three income levels. We can see that if Olemooans at first have a middle income (the middle curve), but then their income increases, the entire demand curve shifts (or moves) to the right, to the curve labelled 'high income'. This curve tells us that for any particular price, Olemooans will buy more pizzas with a higher income, and is called 'an increase in demand'. If, however, Olemooans' income falls, their demand curve will shift to the left to the curve labelled 'low income'; this is called a 'decrease in demand'. This means that for a particular price, Olemooans will buy fewer pizzas.

Leftward/rightward shifts and upward/downward shifts of the demand curve

We have referred to the demand curve as shifting 'to the right' or 'to the left', which is the customary

Price per kg (Olemooan $)	Quantity bought (thousand pizzas per week) Low income	Quantity bought (thousand pizzas per week) Middle income	Quantity bought (thousand pizzas per week) High income
1	8	9	10
2	6	7	8
3	4	5	6
4	2	3	4
5	0	1	2

Table 5: Demand for pizzas by Olemooans at different income levels

practice. However, if you examine Figure 8, you will notice that a rightward shift looks the same as an upward shift, and a leftward shift looks exactly the same as a downward shift. *The meaning of a rightward shift of a demand curve is exactly the same as an upward shift, and the meaning of a leftward shift of a demand curve is the same as a downward shift*, although there is a difference in how we can interpret them.

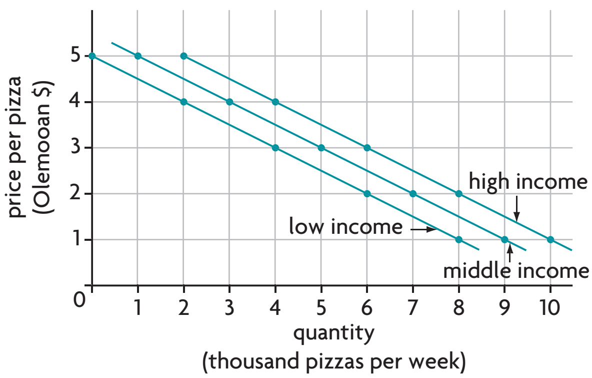


Figure 8: Demand curves at different income levels

If we view the curve as shifting to the right, we see that for a given price, Oloemooans increase their purchases of pizzas as income rises. At a price of $3, they will buy 4000 pizzas per week with a low income, 5000 pizzas with a middle income, and 6000 pizzas with a high income. If we view the curve as moving upward or downward, we see how much Olemooans are willing to pay for a particular quantity of pizzas as their income changes. For example, for the quantity of 6000 pizzas, they are willing to pay $2 per pizza with low incomes, $2.50 per pizza with medium incomes, and $3.00 per pizza with high incomes.

Usually, when studying demand curves, we examine shifts as occurring in the rightward or leftward directions, and less often in the upward and downward directions.

Distinguishing between a movement along the demand curve and a shift of the curve

It is very important to make a distinction between **a movement along a demand curve**, and **a shift of a demand curve**. In a causal relationship, a movement along a curve can only be caused by a change in the independent variable (in this case price), which influences the dependent variable (quantity demanded), thus causing a movement from one point on the curve to another. A shift of a curve, on the other hand, is caused by a change in a variable that was previously held constant under the *ceteris paribus* assumption (the level of income). All variables that can cause shifts of a demand curve are referred to as **determinants of demand**, because they *determine* the position of the demand curve. As you will learn in Chapter 2 of the coursebook, there are several important determinants of demand, of which income is only one.

Supply

Supply as a relationship between price and quantity supplied

Supply involves the relationship between the price of a good and the quantity of the good producers (or firms) want to produce and sell. Table 6 shows the quantity of pizzas Olemooan pizza producers want to produce each week at different possible prices (in Olemooan $). The data in Table 6 are graphed in Figure 9 which shows that there is a positive (or direct) relationship between price and quantity: the higher the price of pizzas, the more pizzas that Olemooan pizza producers want to sell. The quantity of pizzas Olemooan producers want to sell is called **quantity supplied**.

As in the case of demand, price is the independent variable and quantity supplied is the dependent variable, because the quantity supplied depends on price. There is therefore a positive causal relationship between price and quantity supplied. Again, as in the case of demand, *supply curves measure the independent variable, price, on the vertical axis and the dependent variable, quantity, on the horizontal axis.*

Shifts of the supply curve and the ceteris paribus assumption

The amount of a good produced, such as pizzas, depends on more factors than just price. For example, it depends on the number of pizza producers; as this number increases, the amount of pizzas produced will also increase. In addition, it depends on the costs of producing pizzas; if costs increase, it is likely that the amount of pizzas produced will fall.[1]

Price per pizza (Olemooan $)	Quantity supplied (thousand pizzas per week)
1	3
2	4
3	5
4	6
5	7

Table 6: Supply of pizzas by Olemooan producers

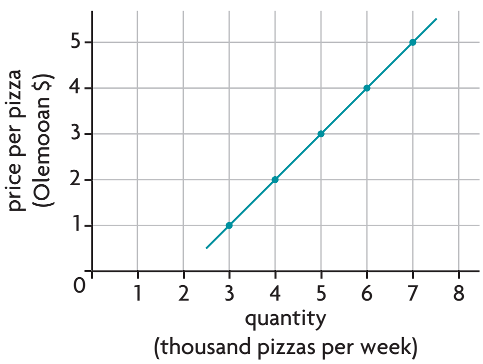


Figure 9: Price of pizzas and quantity supplied

Once again, as in the case of demand, we have more than two variables to deal with, but with the possibility of showing only two of them at the same time in a single supply curve. To address this problem we use the same method as with demand, which involves use of the *ceteris paribus* assumption. We plot the relationship between price and quantity supplied, on the assumption that all other variables that can influence the amount supplied are constant or unchanging.

We can use the information in Table 7 to show what happens to the supply curve when there are factors other than price that affect the amount supplied. The factor we will consider is the cost of producing pizzas. The data in the column indicating 'medium costs' are the same as those we used to plot the supply curve of Figure 9.

Figure 10 graphs the supply curves corresponding to each of the three cost levels. If Olemooan producers have medium costs of production for pizzas and then these costs decrease, the entire supply curve shifts to the right, to the curve indicating 'low costs'. This is called an 'increase in supply', and tells us that at each possible price, producers will supply more pizzas than before. If, however, costs of producing pizzas increase, then the supply curve will shift to the left to the curve labelled 'high costs', meaning that at each possible price producers want to supply fewer pizzas than before.

Leftward/rightward shifts and upward/downward shifts of the supply curve

Looking at Figure 10, we can see that a rightward shift of the supply curve looks the same as a downward shift, and a leftward shift looks the same as an upward shift. The meaning of a rightward shift of a supply curve is exactly the same as a downward shift, and the meaning of a leftward shift of a supply curve is exactly the same as an upward shift, though it is possible to interpret them differently (as in the case of demand).

Viewing the curve move from left to right, we see that for each price, pizza producers want to supply more pizzas as their costs fall. At a price of $3, they will supply 4000 pizzas when costs are high, 5000 when costs are medium, and 6000 when costs are low. Viewing the curve move upward, we see what price producers are willing to accept for pizzas as their costs change. To produce 5000 pizzas, for example, they will want to accept a price of $2 if costs are low, $3 if costs are medium, and $4 if costs are high.

This is reasonable, because the higher the costs of making pizzas, the higher must be the price to make it worthwhile for producers to produce the pizzas.

Supply curve shifts are sometimes examined as moves to the left or to the right (see Chapter 2 of the coursebook) and sometimes as moves up or down

Price per pizza (Olemooan $)	Quantity supplied (thousand pizzas per week) Low costs	Quantity supplied (thousand pizzas per week) Medium costs	Quantity supplied (thousand pizzas per week) High costs
1	4	3	2
2	5	4	3
3	6	5	4
4	7	6	5
5	8	7	6

Table 7: Supply of pizzas by Olemooan producers at different cost levels

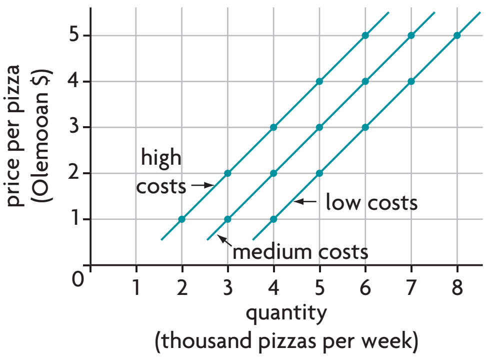


Figure 10: Supply curves at different cost levels

(coursebook Chapters 4–6). However, it is important to note that when we speak of 'an increase in supply' or 'a decrease in supply' we are always referring to leftward/ rightward shifts. An increase in supply means that more pizzas are sold at each price, while a decrease in supply refers to fewer pizzas sold at each price. Since we measure the amount of pizzas along the horizontal axis, this means that supply increases or decreases involve rightward or leftward shifts of the supply curve

Why, then, do we sometimes refer to upward and downward shifts of the supply curve? As you will discover when you study Chapters 4, 5 and 6, upward/ downward shifts are very convenient to use when studying certain kinds of taxes and subsidies, and their effects on firms' supply curves. Taxes work to increase firms' costs of production, and as we know this results in a leftward shift of the supply curve (or a 'decrease in supply'). But *if we view this leftward shift as an upward shift, then it is actually possible to measure the cost increase in our graph*, which can be very useful. Subsidies, on the other hand, are payments by the government to firms, having the opposite effects of taxes. They work to decrease firms' costs of production, resulting in a rightward shift of the supply curve (or an 'increase in supply'). *If we view this rightward shift as a downward shift, we can again measure the decrease in production costs*, which can also be very useful.

These points will become clearer to you as you study the coursebook.

Distinguishing between a movement along the supply curve and a shift of the curve

Everything that was said above in connection with shifts versus movements along a demand curve applies equally to the supply curve. A movement along a supply curve is caused only by changes in price, the independent variable. A shift of a supply curve is caused by changes in variables previously held constant under the *ceteris paribus* assumption. All such variables are called **determinants of supply**, because they determine the position of the supply curve. In Chapter 2, you will learn about several different kinds of determinants of supply.

This leads us to an important point that applies to both demand and supply curves (as well as all other curves):

> A shift of a curve can be caused only by changes in variables that *do not appear* on the vertical or horizontal axis of a graph. Such variables are called *determinants* of the curve, because they determine the position of the curve on the graph. Determinants include all the variables that are held constant under the *ceteris paribus* assumption. If any determinant changes, then the entire curve shifts. On the other hand, any change caused by a variable that is plotted on the vertical or horizontal axis, always leads to a movement along the curve.

TEST YOUR UNDERSTANDING 5

1 Draw a demand curve for pizzas (it is not necessary to use data) and show what is likely to happen if:

- a there is a change in the price of pizzas,
- b there is an increase in the population of Olemoo, and
- c there is a decrease in the population of Olemoo.

2 Draw a supply curve for pizzas (it is not necessary to use data) and show what is likely to happen if

- a there is a change in the price of pizzas,
- b there is an increase in the number of pizza producers, and
- c there is a decrease in the number of pizza producers.

Further topics on graphs

Illustrating two variables that are not related to each other

Sometimes, we may run into variables that are not related to each other. This means that as one variable changes, the other variable remains the same. Two such relationships are shown in Figure 11. In part (a), we see that as fruit consumption increases, the number of haircuts per year remains constant, i.e. fruit consumption does not have any effect on the number of haircuts. In part (b) we see that the number of pizzas bought per person each week remains constant as the price of textbooks changes; in other words, textbook price changes have no effect on pizza consumption. In both these examples, we say the two variables are *independent of each other*.

Calculating areas in a graph

In some situations, some areas in a graph may have a particular meaning that we may want to calculate. For example, an important concept in economics is total revenue (abbreviated as TR), which is the total amount of income that a firm receives for selling its output. Total revenue is calculated by multiplying the number of items of a good sold, or quantity (Q), by the price (P) of the good. Therefore, $TR = P \times Q$. We can see this graphically in Figure 12, which plots price on the vertical axis and quantity on the horizontal axis. The curve shown is a supply curve (S), indicating that the quantity of a good sold by the firm increases as the price of the

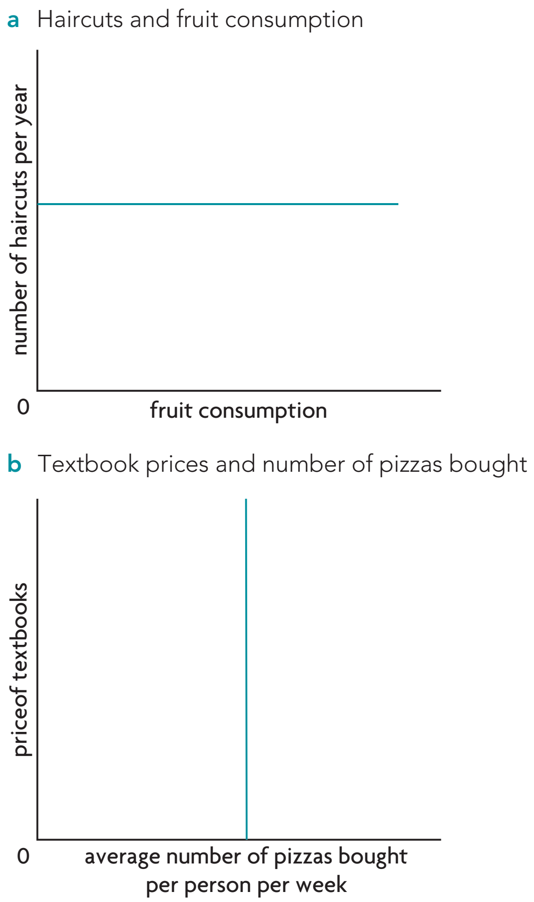


Figure 11: Unrelated variables

good increases. We can see that when price is €2 per unit, the firm sells 30 units; when the price rises to €3 per unit, the firm sells 50 units. What is the firm's total revenue (TR) at each price? Since $TR = P \times Q$, when price is €2, TR = €2×30 = €60, and when price is €3, TR = €3×50 = €150. These values for TR can be shown graphically as areas. Since the area of a rectangle can be found by multiplying two of its adjacent sides (two sides that connect at a point), it follows that TR = €60 is shown by the light green rectangle, and TR = €150 is shown by the sum of the dark plus light green shaded regions. Note that the dark green area alone is the difference between the two, or €90 (= €150 − €60).

TEST YOUR UNDERSTANDING 6

In Figure 12, assume that the price increases to €4 per unit. **a** What quantity of output will be sold at that price? **b** What is the firm's total revenue at the new price? **c** By how much has the firm's total revenue increased compared to when the price was €3 per unit?

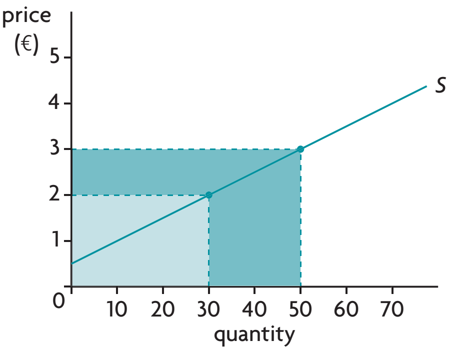


Figure 12: Calculating areas in a graph

Graphs and diagrams in relation to theories and models

The terms 'graph' and 'diagram' are often used interchangeably, and although their meanings certainly overlap, they are not identical. 'Diagram' is a broader term than 'graph', as all graphs are diagrams, yet not all diagrams are graphs. A diagram is any two-dimensional representation of information, which may be non-numerical, i.e. may not involve numbers (although there are many exceptions). A graph, in a most general sense, is a type of diagram that usually displays variables using numbers or quantities (though here, too, there are many exceptions). Graphs are extremely useful in presenting and analysing numerical information, or statistics or data.

Many of the figures presented above consist of diagrams that are graphs, as they all present variables that take on numerical values. On the other hand, the diagram in Figure 11 would usually not be called a 'graph', because it presents information about variables in a non-numerical way.

In economics, when we discuss theories and use models to illustrate theories, we make very heavy use of diagrams that show relationships between variables, but without the use of numbers to measure different values of the variables. In fact, it is not necessary to plot data to show how variables relate to each other. For example, the relationship between price (P) and quantity supplied (Q) could be redrawn as in Figure 13(a); the relationship between price (P) and quantity demanded (Q) could be shown as in Figure 13(b). Neither of these diagrams presents any numerical information, yet we can still immediately see the positive relationship in the first diagram and the negative relationship in the second. Most often, the shape or steepness of curves, either on their own, or drawn together with other curves in the same diagram, are all we need in order to be able to make use of the information they provide to develop theories and illustrate them with models.

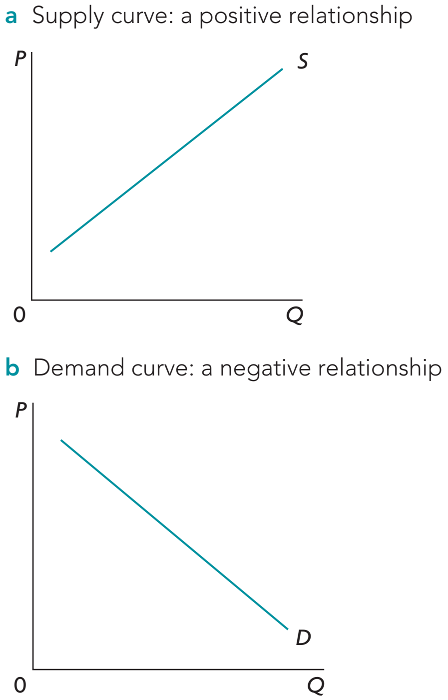


Figure 13: Non-numerical diagrams showing relationships between variables